NON-LINEAR ANALYSIS OF AXIALLY LOADED PILES USING "t-z" and "q-z" CURVES

E. G. Psaroudakis* — MSc Civil Engineer, Department of Civil Engineering, Democritus University of Thrace (DUTh), Greece
Post address: Thermopilon 45, P.K.:54248, Thessaloniki, Greece
E-mail address: empsaroudakis@gmail.com
ORCID: 0000-0002-8540-6127

G. E. Mylonakis — Professor, Chair Geotechnics and Soil-Structure Interaction, University of Bristol, UK
Post address: Queen's Building, University Walk, Clifton BS8 1TR, Bristol, UK
E-mail address: g.mylonakis@bristol.ac.uk
ORCID: 0000-0002-8455-8946

N. S. Klimis — Associate Professor, Department of Civil Engineering, Democritus University of Thrace (DUTh), Greece
Post address: Building B', University Campus of Xanthi - Kimmeria, 67100 Xanthi, Greece
E-mail address: nklimis@civil.duth.gr

*Corresponding author

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ABSTRACT: In the present work, the behaviour of a single pile submitted to axial loading is analyzed. Namely, we examine the static stiffness coefficient at the head of a flexible pile, vertically embedded in a homogeneous or multilayer soil of random geometry and mechanical properties. To solve the problem, an analytical closed form solution is developed, based on Winkler's theory. The model is used in combination with suitable shape functions, which describe reliably the vertical movement of the pile with depth. By choosing the appropriate shape functions along with "t-z" and "q-z" curves and following an iterative process, a relatively accurate estimation of the vertical displacement at the head of the pile can be achieved. Unlike traditional numerical solutions, the proposed method does not require discretization of the pile

into finite elements (and afterwards resolution of a system of linear equations of high order) but only discretization in sections aiming integration with depth.

1. INTRODUCTION

The soil-pile interaction problem has attracted significant research interest over the last decades. A satisfactory approximation of the behavior of an axially loaded pile in inhomogeneous soil is the Winkler model. This model, albeit approximate, is widely used and provides satisfactory results, comparable to more precise methods, such as, finite or boundary elements, whereas it is considered to be also a user-friendly method. The fundamental issue in the implementation, is the determination of the Winkler modulus. The methods that have been developed for that purpose, can be divided into three categories: (a) experimental methods (e.g. Coyle & Reese, 1966), (b) strictly mathematical solutions (e.g. Thomas, 1980, Sanchez-Salinero, 1982), (c) theoretical models (e.g. Novak, 1974, Randolph & Wroth, 1978). Regarding the methods of the latter category, a simple, reliable model, capable of providing improved results, for stiffness of the Winkler springs, so that it can be used in practice, is the most desirable solution.

As per the non-linear elastic theory, an analytical solution based on Winkler theory is developed. In this particular model, the mechanical behavior of the soil is simulated via non-linear springs "t-z" and "q-z", placed along the axis and at the base of the pile respectively. The above non-linear springs are combined with shape functions which describe the modification of the vertical displacement of the pile with the depth. By choosing the appropriate shape functions along with "t-z" and "q-z" curves and following an iterative process, a relatively accurate estimation of the vertical displacement at the head of the pile can be achieved. Unlike traditional numerical solutions, the proposed method does not require discretization of the pile into finite elements (and afterwards resolution of a system of linear equations of high order) but only discretization in sections aiming integration with depth.

2. PROBLEM DESCRIPTION

The model under examination, consists of an axially loaded pile of length L, a section area A and *a* modulus of elasticity E_p. The pile is considered to be embedded in a homogeneous or multilayer soil of irregular geometry and random mechanical properties, which is characterized by a variable modulus of elasticity $E_s(x)$, whilst at the pile head an axial force P is applied, as shown in Figure 1:

Fig 1. Model of single pile embedded in an inhomogeneous soil

3. DEVELOPMENT OF THE MODEL

3.1 Homogeneous elastic soil

The differential equation that describes the behavior of an axially loaded single pile on elastic Winkler foundation is the following (Scott, 1981):

$$-E_p A \frac{d^2 u(x)}{dx^2} + k_s\, u(x) = 0 \tag{1}$$

where $u(x)$ is the axial displacement of the pile at x depth, k_s, the modulus of Winkler springs (dimensions: force / square length), E_p, the modulus of elasticity of the pile's material and A, the section of the pile, which remains constant with depth. Equation (1) can be transformed into differential equation (2):

$$\frac{d^2 u(x)}{dx^2} - \lambda^2\, u(x) = 0 \tag{2}$$

Constant λ has dimensions [1 / length] and describes the decrease of pile's settlement with depth. In the case of homogeneous soil with depth it is given by equation (3) (Foi (2014), Psaroudakis et al (2014)):

$$\lambda = \sqrt{\frac{k_s}{E_p A}} \tag{3}$$

The group of solutions of equation (2) are of the following form:

$$u(x) = C_1 \cosh \lambda x + C_2 \sinh \lambda x \tag{4}$$

where C_1 and C_2 are constants depending on boundary conditions, at the base and at the head of the pile respectively.

Through equilibrium over a small section of the pile, the axial load – displacement relationship is given by the equations below

$$P(x) = A\sigma(x) \tag{5}$$

$$\sigma(x) = E_p \, \varepsilon(x) \tag{6}$$

$$\varepsilon(x) = -\frac{\partial u(x)}{\partial x} \tag{7}$$

where $P(x)$ is the axial load at each depth, minus (-) represents the downward movement and ε and σ are respectively the strain and the stress at the pile section. By substituting equations (7) and (6) to equation (5) we get the axial equilibrium of the pile given by the following equation:

$$P(x) = -E_p A \frac{\partial u(x)}{\partial x} \tag{8}$$

Solving this equation for $x=0$ (pile head) we obtain:

$$P = P(0) = -E_p A \left.\frac{\partial u(x)}{\partial x}\right|_{x=0} \overset{(4)}{=} -E_p A C_2 \lambda \Rightarrow$$

$$C_2 = -\frac{P}{E_p A \lambda} \tag{9}$$

whilst, solving again equation (8) for $x=L$ (pile tip) we obtain:

$$P_b = P(L) = -E_p A \left.\frac{\partial u(x)}{\partial x}\right|_{x=L} = K_b \left.\frac{\partial u(x)}{\partial x}\right|_{x=L} = K_b \, \lambda \left[C_1 \sinh \lambda L + C_2 \cosh \lambda L\right] \overset{(4)}{\Rightarrow}$$

$$C_1 = \frac{P}{E_p A \lambda} \frac{1 + \dfrac{K_b}{E_p A \lambda} \tanh \lambda L}{\dfrac{K_b}{E_p A \lambda} + \tanh \lambda L} \tag{10}$$

where, K_b is the stiffness of the spring at the base of the pile. Mylonakis and Gazetas (1998) used the aforementioned approach to determine the stiffness of an individual pile in a homogeneous soil.

The term $\frac{K_b}{E_p A \lambda}$ reflects the stiffness at the pile base and is replaced by Ω. Consequently, equation (10) is simplified as follows:

$$C_1 = \frac{P}{E_p A \lambda} \frac{1 + \Omega \tanh \lambda L}{\Omega + \tanh \lambda L} \qquad (11)$$

By substitution of C_1 and C_2 in equation (4) we obtain:

$$u(x) = \frac{P}{E_p A \lambda} \left[\frac{1 + \Omega \tanh \lambda L}{\Omega + \tanh \lambda L} \cosh \lambda x - \sinh \lambda x \right] \qquad (12)$$

which describes the vertical displacement of the pile across its length. As stiffness of the pile head, is defined by:

$$K = \frac{P}{u_o} \qquad (13)$$

where $u_o = u(0)$, it results that for axial movement of a pile head embedded into a homogeneous soil, stiffness can be obtained by the following equation:

$$K = E_p A \lambda \frac{\Omega + \tanh \lambda L}{1 + \Omega \tanh \lambda L} \qquad (14)$$

Analytical solutions for homogeneous elastic soil, have been also proposed by Randolph and Worth (1978) and Mylonakis (1995). In Figure 2 results of the aforementioned solution and the one proposed by Mylonakis (1995), are depicted. The horizontal axis represents the normalized pile length, whereas the vertical axis denotes the normalized pile stiffness. As it can be seen, results for homogeneous soil profile and different values of stiffness at the base of the pile (Ω), are almost identical.

Fig 2. illustrates a comparison of the results arisen from the proposed method and those predicted by Mylonakis (1995)

3.2 Inhomogeneous elastic soil

In case of an inhomogeneous elastic soil profile, equation (1) becomes:

$$-E_p A \frac{d^2 u(x)}{dx^2} + k_s(x) u(x) = 0 \tag{15}$$

For stiffness $k_s(x)$ varying with depth, numerical solutions have been proposed by Randolph & Wroth (1978) and Rajapakse (1990). Those solutions are rather complicated and take into account stiffness linearly increasing with depth.

Solution of equation 15 is succeeded by an approximate method including a combined use of an iterative process together with a suitable shape function, $\psi(x)$, which describes reliably, the axial displacement of the pile with depth.

This function is obtained, once, the parameters C_1 and C_2, are calculated through boundary conditions at the head and at the base of the pile, for unit displacement at its head and a ψ_b displacement at its base. By implementation of the aforementioned method, we get λ, varying with depth and consequently it is calculated via the following equation:

$$\lambda(x) = \sqrt{\frac{k_s(x)}{E_p A}} \tag{16}$$

Thus, the group of the solutions of equation (15) are described as follows:

$$u(x) = C_1 \cosh \lambda(x) x + C_2 \sinh \lambda(x) x \tag{17}$$

In order to obtain the shape function $\psi(x)$, C_1 and C_2 need to be determined. First, by setting a unit displacement at the pile head we obtain:

$$u(0) = 1 \Rightarrow C_1 = 1 \tag{18}$$

then, by setting ψ_b displacement at the pile tip, we obtain:

$$u(L) = \psi_b \Rightarrow C_2 = \frac{\psi_b - \cosh \lambda(x) L}{\sinh \lambda(x) L} \tag{19}$$

Substituting C_1 and C_2 in equation (17) and bearing in mind, that:

$$\sinh \lambda(x) x = \frac{e^{\lambda(x) x} - e^{-\lambda(x) x}}{2} \tag{20}$$

and

$$\cosh \lambda(x) x = \frac{e^{\lambda(x) x} + e^{-\lambda(x) x}}{2} \tag{21}$$

we achieve to get the new shape function:

$$u(x) = \psi(x) = \frac{(\psi_b - e^{-\lambda(x) L}) e^{\lambda(x) x} + (e^{\lambda(x) L} - \psi_b) e^{-\lambda(x) x}}{e^{\lambda(x) L} - e^{-\lambda(x) L}} \tag{22}$$

By replacing $\lambda(x)$ with μ, defined as the shape parameter equal to the average value of $\lambda(x)$ along the pile:

$$\mu = \frac{1}{L} \int_0^L \lambda(x)\, dx \tag{23}$$

the shape function $\psi(x)$ is simplified, as following:

$$\psi(x) = \frac{(\psi_b - e^{-\mu L}) e^{\mu x} + (e^{\mu L} - \psi_b) e^{-\mu x}}{e^{\mu L} - e^{-\mu L}} \tag{24}$$

Assuming that the axial displacement of the pile head is equal to unit, then parameter ψ_b is defined as the ratio of the displacement at the base to the displacement at the head of the pile (Figure 3)

Fig 3. [A] Pile model with springs along the side surface and at the base of the pile,

[B] Shape function of pile settlement, $\psi(x)$

Multiplying the approximate unit shape function by the real displacement of the pile head, we calculate the axial displacement along depth $u(x)$:

$$u(x) = u_o\, \psi(x) \tag{25}$$

and consequently, we obtain at the tip:

$$P_b = K_b\, \psi_b = K_b\, u_o\, \psi(L) \tag{26}$$

where $P_b = P(L)$, is the axial load at the pile tip.

Assuming that a theoretical approximate function – solution ($g(x)$) describes the axial displacement of the pile, this function should satisfy the following conditions:

- be at least once differentiable
- fulfil the boundary conditions

Multiplying equation (15) by the approximate function - solution $g(x)$ and then integrating along the pile, we get equation (27):

$$\int_0^L E_p\, A \frac{d^2u(x)}{dx^2}\, g(x)\, dx + \int_0^L k_s(x) u(x) g(x)\, dx = 0 \tag{27}$$

Integrating by parts the first term of the later equation, it is transformed into a simpler form:

$$\int_0^L - E_p\, A \frac{d^2u(x)}{dx^2}\, g(x)\, dx = - E_p\, A \frac{du(x)}{dx}\, g(x)\Big|_{x=0}^{x=L} + \int_0^L E_p\, A \frac{du(x)}{dx} \frac{dg(x)}{dx}\, dx \overset{(8)}{=} \tag{28}$$

$$= P(L)\, g(L) - P(0)\, g(0) + \int_0^L E_p\, A \frac{du(x)}{dx} \frac{dg(x)}{dx}\, dx$$

and substituting into equation (27), it becomes:

$$P(L)\,g(L)-P(0)\,g(0)+\int_0^L E_p\,A\frac{du(x)}{dx}\frac{dg(x)}{dx}dx+\int_0^L k_s(x)u(x)g(x)\,dx\;=0 \tag{29}$$

Now, assuming that:

$$g(x)=u(x)=u_o\,\psi(x)=u(0)\psi(x) \tag{30}$$

and

$$P(L)=P_b=K_b\,u(L)=K_b\,u_o\,\psi(L)=K_b u(0)\psi(L) \tag{31}$$

and by substituting equations (30) and (31) into (29), we finally get:

$$P(L)\,g(L)+\int_0^L E_p\,A\frac{du(x)}{dx}\frac{dg(x)}{dx}dx+\int_0^L k_s(x)u(x)g(x)\,dx\;=P(0)\,g(0)\Rightarrow$$

$$K_b\,u_o\,\psi_b\,u_o\,\psi_b+\int_0^L E_p\,A\frac{du(x)}{dx}\frac{dg(x)}{dx}dx+\int_0^L k_s(x)u(x)g(x)\,dx\;=P\,u_o\Rightarrow$$

$$K_b\,u_o\,\psi_b\,u_o\,\psi_b+\int_0^L E_p\,A\frac{du(x)}{dx}\frac{dg(x)}{dx}dx+\int_0^L k_s(x)u(x)g(x)\,dx\;=P\,u_o\Rightarrow$$

$$K_b\,u_o^2\,\psi_b^2+\int_0^L E_p\,Au_o^2\,\frac{d^2\psi(x)}{dx^2}dx+\int_0^L k_s(x)u_o^2\left[\psi(x)\right]^2dx\;=P\,u_o\Rightarrow$$

$$K_b\,u_o\,\psi_b^2+\int_0^L E_p\,Au_o\,\frac{d^2\psi(x)}{dx^2}dx+\int_0^L k_s(x)u_o\left[\psi(x)\right]^2dx\;=P\Rightarrow$$

$$\left[K_b\;\psi_b^2+\int_0^L E_p\,A\frac{d^2\psi(x)}{dx^2}dx+\int_0^L k_s(x)\left[\psi(x)\right]^2dx\;\right]u_o=P \tag{32}$$

Considering that sinking (axial displacement) at the pile head, is:

$$u_o=\frac{P}{K} \tag{33}$$

the value of stiffness for axial displacement of the pile head is obtained as follows:

$$K = \int_0^L E_p\, A\left[\psi'(x)\right]^2 dx + \int_0^L k_s(x)\left[\psi(x)\right]^2 dx + K_b\, \psi_b^{\ 2} \qquad (34)$$

In equation (34), the first part stands for the stiffness of the pile material, the second one, represents the stiffness due to the resistance along the pile shaft, whilst the third one is the stiffness at the pile base.

Once spring stiffness along the pile $k_s(x)$ and at the pile tip (K_b) are known, by setting the stiffness of the pile section (E_pA) and the axial load at the pile head P, then the axial displacement of the pile u_o is obtained by iterative implementation of equation (32). At the first iteration of the aforementioned process, a random parameter ψ_b ($0<\psi_b<1$) is used. For the forthcoming iterations, the value of the parameter, ψ_b, is obtained from the previous iteration. Convergence is generally achieved in less than ten (10) iterations, whereas the results are in good agreement with rigorous numerical solutions, such as those proposed by Randolph & Wroth (1978) and Rajapakse (1990).

In order to check the reliability of the proposed method, we consider an inhomogeneous elastic soil profile whose stiffness is described by equation 35:

$$k_s(x) = k_s(L)\left[\alpha + (1-\alpha)\frac{x}{L}\right]^n \qquad (35)$$

where,

$$\alpha = \left[\frac{k_s(0)}{k_s(L)}\right]^{1/n} \qquad (36)$$

and $k_s(0)$ is the stiffness at the soil surface, $k_s(L)$ is the soil stiffness at the pile tip and n is a parameter dependent on soil stiffness along the pile, as seen in Figure 4.

Fig 4. Increase of soil stiffness with depth

Given the aforementioned soil profile for parameter $n=1.0$ and α ranging between 0.2 to 0.6, comparisons between results coming from the proposed herein method with rigorous numerical solutions are depicted in normalized diagrams (Figures 5a-c).

Fig 5. Comparison of stiffness at the pile head for inhomogeneous soil profiles

In those diagrams (Figures 5a-c), the stiffness of the pile at the head, is normalized by the side shear modulus at the pile tip (G_{sL}) and subsequently multiplied by the pile radius (r). As it can be seen, results are in good agreement with existing rigorous numerical solutions, with a maximum deviation less than 14%.

4. NON-LINEARITY OF THE SOIL

In order to simulate soil behavior at small axial displacements, curves "t-z" and "q-z" are applied respectively at the side surface and at the base of the pile. These curves describe the relation of soil stiffness, $k_s(x)$ and K_b, with the depth and the axial displacement of the pile, introducing thus into the solution the non-linear behavior of the soil material. Those curves resulted either experimentally, by analyzing load tests on a full scale, or analytically based on a theoretical model.

Among the most popular curves we can cite those of Seed & Reese (1957), Randolph & Wroth (1978), Kraft et al (1981), Fahey & Carter (1993), Bustamante & Frank (1997), as well as, the one proposed by the API (American Petroleum Institute, 1993) based on a large number of experimental data.

A standard curve is presented in Figure 6. The initial part of this curve consists of a rectilinear section which describes soil behavior for small scale displacements (elastic behavior). The stiffness (k_s) of the spring in this part is defined as $\delta \times E_s$ where δ is a dimensionless constant with a mean representative value of 0.5. The final (horizontal) part of the curve, defines the maximum (ultimate) soil reaction, which remains constant in this section, irrespective of the increasing axial displacement of the pile.

Fig 6. A typical t-z curve

In Figures 7 & 8 normalized "t-z" and "q-z" curves proposed by different researchers (Randolph & Wroth (1978), Kraft et al (1981), Fahey & Carter (1993) and API (1993)) are presented. All curves are calculated for a depth of $10m$, soil unit weight $\gamma=20kN/m^3$, friction angle $\varphi=30^0$, pile diameter $d=1.0m$, soil Poisson ratio $v=0.33$, friction parameter on the pile-soil surface $a=0.9$ and soil modulus of elasticity at this depth $E_S=20MPa$.

Fig 7. Normalized curve *q-z* at the tip of the pile

Fig 8. Normalized curve *t-z* at the side of the pile

The curves are applied at the pile as shown in Figure 9 and are used in combination with the proposed method in order to introduce the non-linearity of the soil.

Fig 9. Normalized curves *t-z* alongside the pile and *q-z* at the tip

5. ITERATIVE PROCESS METHOD

As already mentioned, the proposed method is applied iteratively in the case of inhomogeneous elastic soil. In the case of an inhomogeneous inelastic profile, the same procedure is followed, with the soil stiffness being adjusted, using the "t-z" and "q-z" curves, according to the axial displacement of the pile, at each depth.
The iterative process is divided into ten (10) individual steps, described as follows (Figure 10):

Fig 10. Flowchart of iterative process

6. RELIABILITY OF THE PROPOSED METHOD

We hereafter, present two case studies where theoretical results from the proposed method are compared to real field data from in-situ pile load tests.

6.1 Case study No 1

We present hereby the results of a static axial load test performed at the bridge Nottoway River in Virginia, USA (2000) and we compare them to the theoretical ones obtained by the suggested method. The cross-section of the pile tested is square with a side $b=0.61m$ (equivalent diameter $d=0.78m$), length $L=18m$ and

$E_pA=8.2x10^6kN$. The underground water table is located at a depth $8.1m$ from the free ground surface, while soil stratigraphy is presented in Table 1.

Table 1. Soil Stratigraphy

Layer 1	*Medium dense sand 1*	$\gamma=20\ kN/m^3,\ \varphi=33^o,\ v=0.29,\ a=0.9$	*0.0 – 4.0m*
Layer 2	*Medium dense sand 2*	$\gamma=20\ kN/m^3,\ \varphi=32^o,\ v=0.29,\ a=0.9$	*4.0 – 8.0m*
Layer 3	*Stiff clay*	$\gamma=20\ kN/m^3,\ \varphi=22^o,\ v=0.35,\ a=0.9$	*8.0–18.0m*

The aforementioned values have been indirectly deduced from SPT tests performed during the geotechnical investigation. The maximum axial load imposed at the pile was 3095kN, while the ultimate load bearing capacity was estimated at 3508kN.

Figure 11 presents the axial load - settlement curve resulting from the proposed method, as compared with the curve obtained from the pile test load. The "t-z" and "q-z" curves used for simulation of the soil, alongside the pile and at its base, are the one proposed by API (1993).

Fig 11. Comparison of theoretical results of the proposed method (API t-z curves) to measured data from in-situ pile test

6.2 Case study No 2

The 2nd case study refers to axial loading field test of a monopile, at Komotini city located in North-Eastern Greece. The cross-section of the pile used in this case study is circular with a diameter $d=0.8m$, length $L=18.5m$ and $E_pA=14x10^6kN$. The underground water table is located at a depth of $3.5m$ from free ground surface, while soil stratigraphy is presented in Table 2.

Table 2. Soil Stratigraphy

Layer 1	*Dense clayey sand with gravels*	γ=20.8 kN/m^3, φ=43^o, v=0.25, a=0.95	*0.0 – 3.5 m*
Layer 2	*Medium dense clayey sand with a few gravels*	γ=20.5 kN/m^3, φ=35^o, v=0.3, a=0.95	*3.5 – 8.0 m*
Layer 3	*Dense clayey sand with a few gravels*	γ=20.7 kN/m^3, φ=36^o, v=0.28, a=0.95	*8.0–14.0 m*
Layer 4	*Medium dense to dense clayey sand with gravels*	γ=21 kN/m^3, φ=39^o, v=0.23, a=0.95	*14.0–18.50 m*

The aforementioned values have been indirectly calculated from SPT tests performed during geotechnical investigation. The maximum axial load imposed at the pile was 4500kN, while the ultimate load bearing capacity, according to DIN 4014/1990, was estimated at 3600kN, as stated in the relevant study (Geognosi, 1995).

Figure 12 presents the axial load - settlement curve resulting from the proposed method, as compared with the curve obtained from the pile test load. The "t-z" and "q-z" curves used for simulation of the soil, alongside the pile and at its base, are those proposed by API (1993).

Fig 12. Comparison of theoretical results of the proposed method (API t-z curves) to measured data from in-situ pile test.

Results of the proposed method, as presented in Figures 11 and 12, are in reasonable agreement with in-situ test results. Namely, in Figure 12 we notice that the proposed solution approximates in a better way field results for the 2nd upcoming branch of loading. Both experimental data and results of the proposed method seem to converge reasonably well for values in the proximity of axial bearing capacity. On the other hand, in Figure 11, the proposed solution fits better field data in the elastic part of the curve, almost up to 70% of the axial bearing capacity, whilst in the proximity of the ultimate axial load (axial bearing capacity), appear to be less conservative compared to the experimental ones.

7. CONCLUSIONS

A simple numerical model has been developed for the analysis of the non-linear problem of an axially loaded monopile without using finite elements or finite differences and consequently resolution of numerous linear systems. The proposed method is iterative and is based on two integrals and on a supplementary term (eq. 34) related to the participation of the pile and the soil to the overall stiffness at the pile-head.

If the soil is simulated by linear springs, then the proposed solution is quite similar to the already existing solutions in bibliography, whereas, in the case of a homogeneous soil, convergence is achieved without iterative process. The validation process of the proposed method over results of full scale tests demonstrated a remarkable agreement.

Moreover, as the proposed method is not based on the laborious process of matrix inversion, the solution remains stable even for axial loads close to failure e.g. pile axial capacity.

Finally, due to the combined advantages, such as, stability, accuracy, economy and ease of use, it is our belief that the proposed method can be reliably applied whenever there is lack of experimental data or as an alternative solution instead of laborious numerical analyses.

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